

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS  
SPECIALIST  
UNITS 3&4

**SOLUTIONS**

Section One:  
Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 49 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 91 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (49 Marks)

This section has**seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

The point and part of the slope field given by is shown below.

<EFOFEX>
id:fxd{d47d5fcf-b370-4f73-833c-9bb713b4acc4}

FXData:

</EFOFEX>

(a) Determine the equation of the solution curve through in the form . (4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ separates variables as an integration statement  ü correctly uses trig identity on RHS  ü correctly obtains antiderivative, with constant  ü evaluates constant and writes in required form |

(b) Draw the solution curve through on the slope field above. (2 marks)

|  |
| --- |
| Solution |
| See slope field |
| Specific behaviours |
| ✓ curve to left of -axis has two inflection points and follows slope field  ü curve to right of -axis has two inflection points and follows slope field |

Question 2 (7 marks)

Let .

(a) Sketch the graph of . (1 mark)

<EFOFEX>
id:fxd{272c59fc-dc91-45d7-ac6d-49a43757fc0e}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ reasonable sketch |

(b) On the axes below, sketch the graph of . (2 marks)

<EFOFEX>
id:fxd{7c3aa8ab-e384-4bb7-a53f-8dba58993e15}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ symmetry about -axis  ü correct sketch |

Consider .

(c) Briefly describe how the graph of can be used to sketch the graph of and hence state the domain and range of . (4 marks)

|  |
| --- |
| Solution |
| The graph of is the same as the graph of .  Domain .  Range . |
| Specific behaviours |
| ✓ explains is reciprocal of  ü correct restrictions on for domain  ü states component of range  ü states component of range |

Question 3 (8 marks)

The equations of planes and are shown below.

(a) Show that the equation of plane in Cartesian form is . (3 marks)

|  |
| --- |
| Solution |
| Cross product of vectors in plane is .  Let normal to plane be , so that .  Hence Cartesian equation is . |
| Specific behaviours |
| ✓ writes cross product of two vectors that lie in  ü obtains normal to plane  ü uses vector equation to derive Cartesian equation |

(b) The origin lies on the surface of sphere . Determine the vector equation of , given that its centre is the point of intersection of the three planes. (5 marks)

|  |
| --- |
| Solution |
| Subtracting first from middle gives .  Adding first to last gives .  Hence and so intersect at .  Distance from centre of to is .  Hence equation of is . |
| Specific behaviours |
| ✓ writes system of equations, correctly starts elimination  ü solves for one variable  ü solves for all variables  ü determines distance from origin  ü correctly writes equation of sphere |

Question 4 (6 marks)

Let , where and are real constants.

One of the roots of is .

(a) Determine the value of the constant and the value of the constant . (4 marks)

|  |
| --- |
| Solution |
| Product of factors using given root and conjugate:  Hence  Comparing coefficients:  Hence from expansion of factors, and . |
| Specific behaviours |
| ✓ indicates conjugate is another root  ü obtains quadratic factor of  ü uses quadratic factor to obtain linear factor  ü states correct values for and |

(b) Show all the roots of in the complex plane below. (2 marks)

<EFOFEX>
id:fxd{49813c5f-6855-4d93-9298-d927ecb00c5f}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See diagram |
| Specific behaviours |
| ✓ shows given root and conjugate  ü shows real root |

Question 5 (7 marks)

An electronic circuit will remain stable when and , the resistances in ohms of two variable resistors in the circuit, satisfy

(a) When is decreasing at a rate of ohms per second, determine the rate that must be changing for the circuit to remain stable when ohms. (4 marks)

|  |
| --- |
| Solution |
| Require given , so implicitly differentiate wrt :  Hence must increase at ohms per second for stability. |
| Specific behaviours |
| ✓ implicitly differentiates equation  ü obtains expression for  ü indicates correct values for and  ü correctly evaluates |

(b) The circuit is stable, the resistance of is increasing at ohms per second and has just reached ohms. Use the technique of increments to calculate the approximate change in the resistance of in the next tenth of a second. (3 marks)

|  |
| --- |
| Solution |
| Resistance of will decrease by ohm. |
| Specific behaviours |
| ✓ states correct values for and  ü forms correct expression for  ü correct value for |

Question 6 (8 marks)

<EFOFEX>
id:fxd{f8f26b24-c1da-4d14-9eb2-b79913bddd8f}

FXData:

</EFOFEX>(a) The line and the curve are  
shown in the diagram. They intersect at the  
origin and at .  
  
Determine the area between the curve and the  
line in the first quadrant.

(5 marks)

|  |
| --- |
| Solution |
| To determine area under curve let , so that and . Then  Area under line is .  Hence area between is . |
| Specific behaviours |
| ✓ chooses suitable substitution  ü writes integral in terms of  ü obtains antiderivative  ü obtains area under curve  ü obtains area between |

(b) Given that , determine . (3 marks)

|  |
| --- |
| Solution |
| Let , so that , and .  Then |
| Specific behaviours |
| ✓ chooses substitution and relates and  ü writes integral in terms of  ü derives correct value |

Question 7 (7 marks)

(a) Determine the value of the constant and the value of the constant so that

(2 marks)

|  |
| --- |
| Solution |
| Combining RHS into single fraction and equating numerators:  Hence and . |
| Specific behaviours |
| ✓ indicates appropriate method  ü solves for correctly |

(b) Hence use the substitution to determine

(5 marks)

|  |
| --- |
| Solution |
| Simplify denominator  Also  Hence |
| Specific behaviours |
| ✓ simplifies denominator in terms of  ü correctly relates and  ü uses partial fractions to express integral in terms of  ü obtains antiderivative with absolute value brackets  ü correct result in terms of , and integration constant |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

© 2022 WA Exam Papers. Perth Modern School has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN078-204-3.